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A modified BCS theory of heavy-fermion superconductivity

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Abstract. In this paper we derive an expression for the superconducting transition temperature of a uranium-based heavy-fermion system within a modified weak-coupling theory of superconductivity. It is found that the energy dependence of the enhanced density of states for the heavy-fermion system clearly manifests itself in the theory, and the Kondo energy naturally takes the role of cut-off, as long as the effective cut-off energy is large in comparison. The numerical analysis confirms this result and shows that the superconducting transition temperature is indeed independent of the effective cut-off energy employed within the approach.

1. Introduction

The nature of superconductivity in heavy-fermion (HF) systems is still unclear. Although it is generally accepted that pairing of the characteristic heavy quasiparticles must play a crucial role in the superconducting mechanism, the source of this mechanism and the nature of the associated order parameter are not well understood. Of particular interest is the compound UPt_3, which exhibits a phase diagram that is unique [1,2]. The existence of two superconducting phase transitions can only be explained by a theory that postulates an exotic superconducting state.

On the phenomenological side, present theories can be distinguished by two critical ingredients: the symmetry of the Cooper pairs that are formed, whether it be p-wave or d-wave, and the degree of spin–orbit coupling in the system. At the present time, there is no single model that is consistent with all the experimental facts. For a brief review see [3].

A thorough microscopic prescription seems even further from any completion. Microscopic theories applied to these systems, in particular UPt_3, are based on one of two different pairing mechanisms; a conventional quasiparticle–phonon interaction (see for example [4]) or a spin-fluctuation-type process. Recent literature has even suggested that it is the coexistence and interplay of both mechanisms that leads to the anomalous properties displayed in these systems [5]. Both weak-coupling and strong-coupling approaches, containing many different forms of quasiparticle pairing, have been put forward. Of all the exotic microscopic theories that have been proposed, none are ultimately conclusive.

In this paper we show how the energy dependence of the enhanced density of states for a highly correlated f-electron system manifests itself when we apply a simple model of superconductivity. The simplicity of the approach rests on the fact that we treat the interaction, V_{int}, that binds two quasiparticles together, as a phenomenological parameter. Apart from this one input, we essentially apply the weak-coupling approach of the BCS theory [6], and its extension by Balian and Werthamer [7], to a Fermi liquid system in which the composite quasiparticles are anomalously heavy.

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We start by deriving a mean-field quasiparticle Hamiltonian in which we have incorporated a quasiparticle coupling term. This naturally describes the Fermi liquid state of the HF system in terms of two quasiparticle bands and provides the basis to which we apply a BCS-type theory. We first construct a one-band theory based on the lower quasiparticle band, and later consider the inclusion of a term which arises from the upper band. The model applies equally to p- and d-wave pairings. This is solved numerically to give the superconducting transition temperature, T_c, in terms of the quasiparticle interaction, V_l. We then derive an analytic expression for the superconducting transition temperature, employing a simple approximation. This enables us to extract the underlying physics of the problem in a clear way. The theory also does not encompass the effects associated with spin fluctuations between quasiparticles at any stage, thus a potential pair-breaking process is neglected. We find that the final expression for the superconducting transition temperature is composed of two distinct contributions. In the first term the cut-off is provided by the interaction, and the density of states is of the bare (unenhanced) form. The second term contains a cut-off provided by the Kondo energy and the effective density of states is enhanced. The latter term will always be dominant for HF systems. A comparison with the numerical results confirms this picture. One conclusion of our work is that the isotope effect should be zero or very small even if HF superconductivity is phonon mediated.

2. Mean-field theory

Our starting point is the generic model for HF systems; the Anderson lattice model

$$H = \sum_{k\sigma} \epsilon_k c^\dagger_{k\sigma} c_{k\sigma} + E_0 \sum_{im} f^\dagger_{im} f_{im} + U \sum_{im \neq m'} n^f_{im} n^f_{im'} + \sum_{ikm\sigma} (S_{km\sigma} e^{ik\cdot R_i} c^\dagger_{k\sigma} f_{im} + \text{HC}) \qquad (1)$$

where ϵ_k is the dispersion relation for the conduction electrons, E_0 is the bare energy of the impurity electrons, and U is the on-site Coulomb repulsion. $S_{km\sigma}$ is the hybridization term. This is usually written as $V_{km\sigma}$, but for clarity we use an alternative notation so that there is no confusion with the interaction, V, between pairs of quasiparticles. All energies are defined relative to the Fermi surface.

We consider this Hamiltonian in the limit of infinite U so that the occupancy at any given site is, for a uranium HF compound, confined between two and three f electrons. We shall not consider the simpler case appropriate for cerium systems. The introduction of two slave boson fields on every site eliminates the Coulomb interaction in this limit, and creates in return a constraint term incorporated into the Hamiltonian via a Lagrange multiplier [8, 9]. It is with respect to the boson fields that the mean-field approximation is made [10]. In this way, the operators are treated as classical fields, the symmetry of the Hamiltonian is broken, and the constraint becomes a global one. We thus derive the following mean-field Hamiltonian [11]

$$H_{\text{mf}} = \sum_{km} \epsilon_{km} c^\dagger_{km} c_{km} + \sum_{im} \tilde{\epsilon}_{\text{f}} f^\dagger_{im} f_{im} + \sum_{im} S(c^\dagger_{im} f_{im} + \text{HC}) + \text{constant} \qquad (2)$$

in which we find renormalized values for the f-level energy, $\tilde{\epsilon}_{\text{f}}$ (shifted above the Fermi level) and hybridization, $S = S_{\text{h}}\sqrt{((n_{\text{f}} - 2)(3 - n_{\text{f}}))}$ (reduced by an order of magnitude) where S_{h} is the bare form assumed to be independent of wave vector and spin.

This expression can be easily diagonalized into a form that clearly represents the quasiparticle energy dispersion

$$H_{\text{qp}} = \sum_{km} E^\alpha_{km} \alpha^\dagger_{km} \alpha_{km} + \sum_{km} E^\beta_{km} \beta^\dagger_{km} \beta_{km} \qquad (3)$$

where E^{α}_{km} and E^{β}_{km} are the lower and upper HF band energies given by

$$E^{\alpha,\beta}(k) = \frac{1}{2}\left(\epsilon_k + \tilde{\epsilon}_f \pm \sqrt{(\epsilon_k - \tilde{\epsilon}_f)^2 + 4S^2}\right) \tag{4}$$

and $\alpha^{\dagger}_{km}$ and $\beta^{\dagger}_{km}$ create lower- and upper-band quasiparticles respectively.

In the temperature range of interest UPt_3 has an antiferromagnetic structure with a small moment. We assume that this can be treated within a band theoretical approach, so that for the very small ordered moment the energy of the polarized bands are hardly changed [13]. Thus for simplicity we shall continue to use equation (4) to describe the quasiparticle energies. This is consistent with the experimental observation that T_c does not vary much when the antiferromagnetism is removed, although the splitting between the two transition temperatures does disappear [14].

We first focus solely on the lower band and consider a general pairing interaction, which couples the quasiparticles so that they form Cooper pairs. The pairing quasiparticle Hamiltonian is then given by

$$H_p = \sum_{km} E^{\alpha}_{km}\alpha^{\dagger}_{km}\alpha_{km} + \sum_{k,k',\sigma,\sigma'} V(\boldsymbol{k},\boldsymbol{k}')\alpha^{\dagger}_{-k\sigma'}\alpha^{\dagger}_{k\sigma}\alpha_{k'\sigma}\alpha_{-k'\sigma'}. \tag{5}$$

The origin of these mechanisms is not considered in the analysis that follows. Although we are constructing a model that employs an interaction over a characteristic energy, which we call Θ, we are not asserting that it is, in fact, a quasiparticle–phonon mechanism. The interaction could arise from paramagnons in which case Θ would be simply a characteristic magnetic energy. The interaction, $V(\boldsymbol{k},\boldsymbol{k}')$ depends on both the magnitude and direction of $\boldsymbol{k}$ and $\boldsymbol{k}'$. It may be expanded in terms of spherical harmonics, and the coefficients, $V_l(|\boldsymbol{k}|,|\boldsymbol{k}'|)$, are separated out in the usual way [15]. We assume that these coefficients are independent of $|\boldsymbol{k}|$ and $|\boldsymbol{k}'|$, both of which are close to k_f. These are then our phenomenological inputs of the theory, which we treat as constant over the given energy range

$$V_l(|\boldsymbol{k}|,|\boldsymbol{k}'|)\begin{cases} \simeq V_l(k_f,k_f) = V_l & |E_{km}| < \Theta \\ = 0 & |E_{km}| > \Theta\,. \end{cases} \tag{6}$$

The pairing of quasiparticles caused by these different types of interaction will ultimately lead to the same form of expression for the transition temperature. In this way, the T_c for p-wave pairing is a function of V_l and correspondingly the T_c for d-wave pairing is a function of V_2. In this paper we will not specify which of these interactions is dominant, but consider a general interaction, V_l, which gives rise to l-wave pairing.

3. Modified BCS theory

In UPt_3, the Kondo temperature (40 K) is much larger than the Neel temperature (5 K) which, in turn, is larger than the superconducting transition temperature (0.5 K). It is reasonable, therefore, to employ a weak-coupling approach. In the usual manner we derive an integral equation which defines the transition temperature, T_c. The crucial point is that the order parameter, defined in terms of V_l, is assumed independent of ϵ_k. It is then constant over the integration and simply cancels. The resulting expression is then very similar to that derived in the standard BCS theory. For the higher-order angular momentum expressions, the angular dependence of the order parameter can be easily dealt with so

that the integral equation becomes a general relation that applies equally to each of the components, $V_0(k, k')$, $V_l(k, k')$ and $V_2(k, k')$[15]

$$\int_{\epsilon_k''}^{\epsilon_{k'}} \frac{\mathrm{d}\epsilon_k}{2(E^\alpha(\epsilon_k) - \mu)} \rho_0 V_l \tanh\left(\frac{\beta_c}{2}[E^\alpha(\epsilon_k) - \mu]\right) = 1 \tag{7}$$

where ρ_0 is the bare density of states, which is taken as constant at the Fermi surface, μ is the chemical potential and β_c is defined in the customary way as $(1/T_c)$. We now change the variable of integration and integrate with respect to the quasiparticle band energy instead of the bare conduction electron energy. This new expression then contains a quasiparticle density of states which is explicitly included

$$\rho_0 V_l \int_{(-\Theta)}^{(\epsilon_m)} \frac{\mathrm{d}E_\mu^\alpha(\epsilon_k)}{2E_\mu^\alpha(\epsilon_k)} \left(1 + \frac{S^2}{(E_\mu^\alpha(\epsilon_k) - \tilde{\epsilon}_f)^2}\right) \tanh\left(\frac{\beta_c}{2} E_\mu^\alpha(\epsilon_k)\right) = 1. \tag{8}$$

Here E_μ^α is simply $E^\alpha(\epsilon_k) - \mu$ and Θ represents the lower cut-off. The upper cut-off however is not Θ, as employed in a conventional BCS-type theory, but ϵ_m which defines the upper edge of the lower quasiparticle band (see figure 1). We are assuming throughout this paper that $\Theta > \epsilon_m$ so that the integral always has this electronic energy for its upper cut-off. ϵ_m can be quite easily expressed in terms of $\tilde{\epsilon}_f$ in the following way

$$\epsilon_m = \tilde{\epsilon}_f - \frac{S^2}{(N-1)W} \simeq \tilde{\epsilon}_f - \frac{\tilde{\epsilon}_f n_f}{(N-1)} \simeq \frac{(N - 1 - n_f)\tilde{\epsilon}_f}{(N-1)} \tag{9}$$

where W is the bandwidth, N the impurity f-level degeneracy and n_f is the impurity site valence, which for uranium HF systems will fluctuate between two and three.

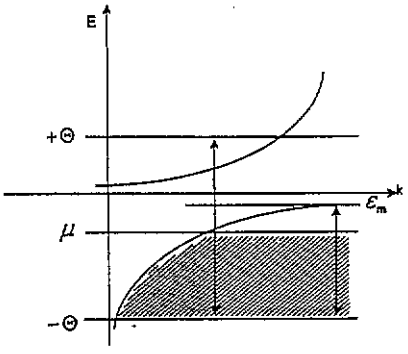

Figure 1. The upper and lower quasiparticle bands showing the two different ranges over which the integration is performed. ϵ_m represents the upper edge of the lower band and Θ is the general cut-off employed. Both are measured relative to μ, the chemical potential.

The choice of limits is more subtle than it at first seems and is crucial in the final analysis. The superconductivity is assumed to be a small perturbation on the HF states and so the interactions should be described in terms of the dressed quasiparticles. The Kondo temperature, T_K, is always found to be the greater energy scale when compared with the superconducting transition temperature, T_c. This must always be the case, because above T_K, the system exhibits classical local moment behaviour. Measurements of the specific heat jump at T_c also imply that it is the heavy quasiparticles that play the central role in HF superconductivity.

We must now deal with the integral given by equation (8). This can be done both numerically and analytically, using a suitable approximation. It is worth noting that the usual analytic approach applied successfully in the conventional BCS theory breaks down for equation (8). We employ a different analytic approach that is workable and consistent with the numerics, and furthermore, reveals the underlying physics of these systems in a clear, natural way.

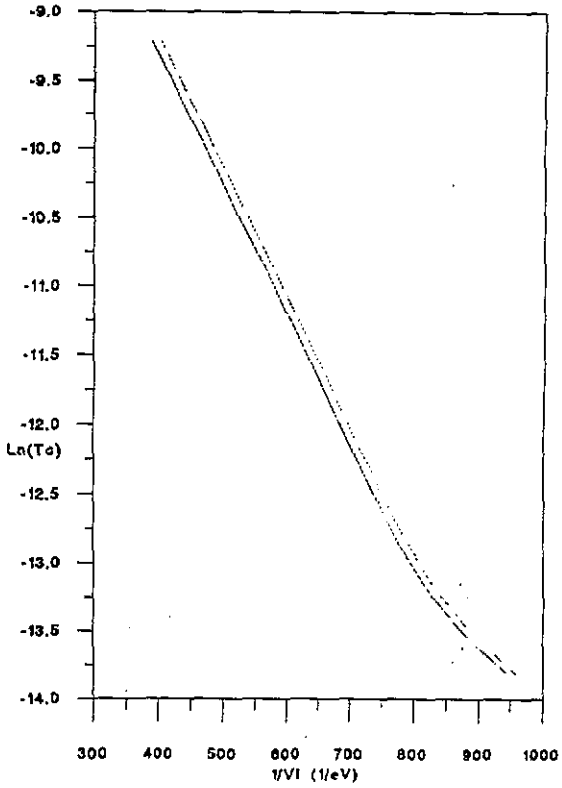

Figure 2. The variation of the transition temperature, T_c, with the pairing potential, V_l. The solid line has been obtained by including excitations within the lower band only while the dashed line represents the full theory in which the upper band is also included. The following parameters were assumed: $W = 10$ eV, $S_h = 1$ eV $N = 6$ and $n_f = 2.85$.

4. The numerical solution

To evaluate the integral in equation (8) numerically, we essentially need five input parameters. These are three energies, the bandwidth, W, the characteristic cut-off energy, Θ, and the bare hybridization, S_h; and two numbers, the f-level degeneracy, N, and the f-level occupancy, n_f. The other relevant energies, the chemical potential, μ and the renormalized f-level energy, $\tilde{\epsilon}_f$, can be derived using the zero-temperature results of the slave-boson mean-field theory [11, 12]. These give the following relations

$$\mu = W(3 - n_f) \tag{10}$$

and

$$\tilde{\epsilon}_f = \frac{S^2}{Wn_f} = \frac{S_h^2(3 - n_f)(n_f - 2)}{Wn_f}. \tag{11}$$

We first take W and S_h as 10 eV and 1 eV respectively. We also assign realistic values to N (= 6) and n_f (= 2.85). A plot of $1/V_l$ against $\ln(T_c)$ with these inputs is shown in figure 2. This reveals a linear behaviour in which there is both a negative gradient and intercept as might be expected. The value of V_l that would correspond to a superconducting transition temperature of 0.5 K is almost 2×10^{-3} eV. If either the bare hybridization or the degeneracy are increased in value, then the corresponding interaction, V_l must be larger for a given T_c, the greater effect occurring for S_h. An increase in the bandwidth, however, requires that V_l be smaller for the same T_c. The most profound effects on the results are felt when the valence is varied between its two limits, two and three. The chemical potential, renormalized hybridization and renormalized f-level energy all depend on what value the

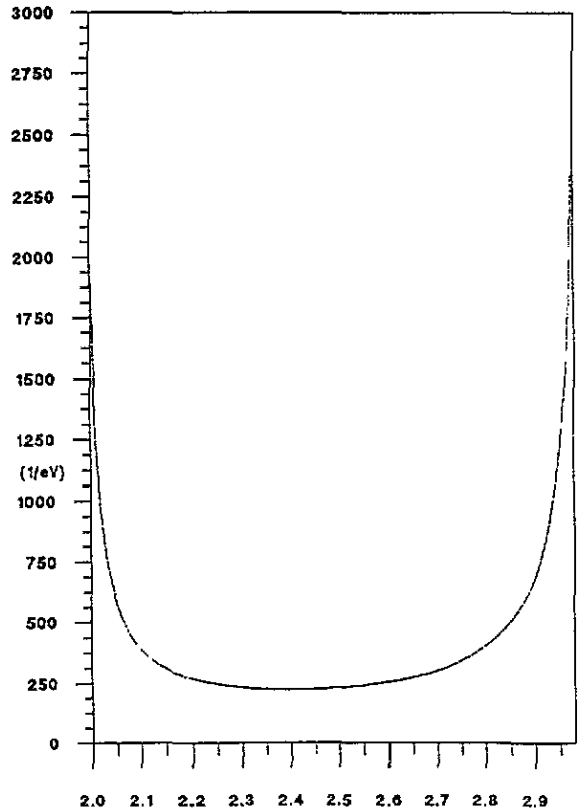

Figure 3. The variation of the coupling, V_l, required to generate a transition to l-wave superconductivity at a transition temperature, $T_c = 0.5$. This figure shows the tendency towards superconductivity is strongly enhanced in the Kondo limits, $n_f \geqslant 2.85$ or $n_f \leqslant 2.1$. In these regions arelatively small value of V_l is required to give the observed value of T_c, with the pairing potential, V_l. The solid line has been obtained by including excitations within the lower band only while the dashed line represents the full theory in which the upper band is also included. The following parameters were assumed: $W = 10$ eV, $S_h = 1$ eV and $N = 6$.

valence holds. Figure 3 shows how the interaction required to yield a transition temperature of 0.5 K varies with n_f. We see that a very small value of V_l is required close to the Kondo limit and a relatively large value for the intermediate region in which $2.2 \leqslant n_f \leqslant 2.7$. The gradient of the plot shown in figure 2 is correspondingly larger for values of n_f close to the Kondo limits.

5. The analytic approach

We find that a simple approximation scheme can reproduce the numerical analysis extremely well. It is also serves to unveil the underlying trends that give rise to these particular results. To proceed analytically with the integral of equation (8) then, we must at some point make an approximation. We do this by replacing the tanh function with an angular form, so that

$\tanh x \simeq$	x
-1	$x \leqslant -1$
x	$-1 < x \leqslant 1$
$+1$	$x > 1$

This also enables us to extract the underlying physics throughout the calculation. It is easy to see that the magnitude of the real tanh function is always less than this angular approximation. This will mean that the value of V_l required to obtain the observed value of T_c will always be too low. We then obtain three trivial integrals

$$I_l = -\int_{-\Theta}^{-2/\beta_c} \frac{dE_\mu^\alpha(\epsilon_k)}{2E_\mu^\alpha(\epsilon_k)} \left(1 + \frac{S^2}{(E_\mu^\alpha(\epsilon_k) - \tilde{\epsilon}_f)^2}\right) \tag{12}$$

$$I_2 = +\frac{\beta_c}{4} \int_{-2/\beta_c}^{+2/\beta_c} dE_\mu^\alpha(\epsilon_k) \left(1 + \frac{S^2}{(E_\mu^\alpha(\epsilon_k) - \tilde{\epsilon}_f)^2}\right) \tag{13}$$

$$I_3 = +\int_{+1/\beta_c}^{\epsilon_m} \frac{dE_\mu^\alpha(\epsilon_k)}{2E_\mu^\alpha(\epsilon_k)} \left(1 + \frac{S^2}{(E_\mu^\alpha(\epsilon_k) - \tilde{\epsilon}_f)^2}\right) \tag{14}$$

Upon evaluation, the terms combine to form the following expression

$$\frac{1}{V_l} = \rho_0 \left(\ln\left|\frac{\beta_c}{2}\Lambda\right| + 1\right) + \rho_* \left(\ln|(\beta_c/2)\Omega| + C\right) \tag{15}$$

where the enhanced density of states is given by $\rho_* = \rho_0(S^2/\tilde{\epsilon}_f)^2$. We have defined Λ as $\sqrt{\Theta\tilde{\epsilon}_f\delta}$ where δ is a degeneracy factor resulting from the energy cut-off at the edge of the lower band. This is equal to $\sqrt{(N-1-n_f)/(N-1)}$. We have also introduced Ω and C which are given by

$$\Omega = \sqrt{\frac{\Theta\delta|(4/\beta_c)^2 - \tilde{\epsilon}_f^2|(N-1)}{(\Theta + \tilde{\epsilon}_f)n_f}} \tag{16}$$

which for $\tilde{\epsilon}_f/kT_c \gg 1$, becomes

$$\Omega = \sqrt{\frac{\Theta\delta\tilde{\epsilon}_f^2(N-1)}{(\Theta + \tilde{\epsilon}_f)n_f}} = \tilde{\epsilon}_f\sqrt{\frac{\delta(N-1)}{n_f(1 + \tilde{\epsilon}_f/\Theta)}} \tag{17}$$

and

$$C = \frac{1}{2}\left(\frac{\tilde{\epsilon}_f}{(\Theta + \tilde{\epsilon}_f)} + \frac{(N-1)}{n_f}\right). \tag{18}$$

It is apparent that this expression is composed of two different forms of contribution. There are those terms which essentially reproduce the conventional BCS result in which Λ takes the place of Θ_D. There are also other terms where the characteristic enhancement factor, $(S^2/\tilde{\epsilon}_f^2)$, comes explicitly into play.

Thus, the general expression is made up of two distinct contributions; one that involves the bare density of states, ρ_0 and the interaction cut-off Θ, and the other which contains the enhanced density of states, ρ_*, and is independent of Θ.

If the enhancement of the density of states characteristic of HF systems was in fact radically reduced then the BCS-type term would eventually dominate

$$T_c = a\Lambda \exp(-1/\rho_0 V_l). \tag{19}$$

For our analytic approximation, a is given by $\exp(1)/2 \simeq 1.36$, compared with the BCS analytic result where $a = 1.134$. In this sense, the result would be very similar to the original BCS result. The energy cut-off, Λ, is given by the geometric mean of the upper and lower cut-offs, Θ and ϵ_m, and is smaller than Θ (for $\epsilon_m < \Theta$). This effective energy cut-off is still, however, relatively large and the density of states is that for the bare conduction electrons.

For HF systems, however, the second contribution to the transition temperature will always dominate and the pairing of highly correlated quasiparticles will prevail

$$T_c = b\Omega \exp(-1/\rho_* V_l) \tag{20}$$

where b is $\exp(C)$, which is approximately unity. Here the cut-off is proportional to the effective Kondo energy, and the density of states is of the enhanced form ($\rho_* = S^2/\tilde{\epsilon}_f^2 \rho_0 = n_f/\tilde{\epsilon}_f$). The interaction, V_l, acts only between heavy quasiparticles in close proximity to the Fermi surface.

The energy Ω is seen to vary linearly with $\tilde{\epsilon}_f$. In this theory we have assumed that $\Theta \geqslant \tilde{\epsilon}_f$ throughout, as shown in figure 1. We note that in the limit $\tilde{\epsilon}_f/\Theta \ll 1$ then Ω is independent of Θ, the interaction cut-off energy. This means that the transition temperature is independent of the cut-off energy, Θ, provided that $\Theta \gg \tilde{\epsilon}_f$. The Kondo energy has taken the role of energy cut-off within the enhanced expression. Hence the exponent α associated with the isotope effect would be zero even if the interactions were phonon mediated in this limit.

The fact that the massively enhanced form of the density of states dominates the expression for T_c is verified by a comparison with the numerical result. A plot of equation (20) containing the same parameters as the numerical plot, yields the same linear dependence. It only differs in that it always slightly overestimates the value of $1/V_l$ for a particular temperature. This is merely the manifestation of the approximation we introduced to evaluate the tanh function (we obtain the same factor if the standard BCS theory is evaluated in this approximation).

6. The upper-band term

So far we have considered a theory that is applicable only to the lower band. A rigorous treatment of both quasiparticle bands would involve a pairing Hamiltonian which would contain all possible types of pairing between quasiparticles. This would also contain several different interactions, V_l, corresponding to particular pairings. We have only considered the scattering of two lower-band quasiparticles, with momenta and spin (k', σ) and $(-k', \sigma)$, into another lower-band pair with momenta and spin (k, σ) and $(-k, \sigma)$. In a two-band theory, other quasiparticle pairings may be significant. In particular, the scattering of two lower-band quasiparticles into two upper-band quasiparticles or one lower-band quasiparticle and one upper-band quasiparticle both might play an important role in a superconducting state. In a later publication we shall examine the two-band theory in its fullest sense, but here, instead, we shall only consider the extension of this theory so that it incorporates, in some way, the former inter-band pairing, that is the scattering of two lower-band quasiparticles into two upper-band quasiparticles.

We do this by simply extending the energy range so that we include a lower portion of the upper band up to the Debye energy (see figure 1). The model thus assumes that the same pairing interaction acts on both the lower-band and upper-band quasiparticles. The

extra term we should include is then of the following form

$$\rho_0 V_l \int_{(2\tilde{\epsilon}_f)}^{(\Theta_D)} \frac{dE_\mu^\beta(\epsilon_k)}{2E_\mu^\beta(\epsilon_k)} \left(1 + \frac{S^2}{(E_\mu^\beta(\epsilon_k) - \tilde{\epsilon}_f)^2}\right) \tanh\left(\frac{\beta_c}{2} E_\mu^\beta(\epsilon_k)\right) = 1. \tag{21}$$

When this is included in the numerical calculation, it causes a very small increase in T_c for a given value of V_l. Apart from this the plot is essentially unchanged. This is shown in figure 2.

It is also interesting to see how this term affects the analytic results. In the same way as before we approximate the tanh function to a linear form. In this energy range it will simply be equal to one (for $T_c \ll T_K$, $2\tilde{\epsilon}_f/k_B T_c \gg 1$), so that we obtain an integral which is almost identical to I_3, differing only in the limits of the integration. We shall call this I_4

$$I_4 = + \int_{2\tilde{\epsilon}_f}^{\Theta} \frac{dE_\mu^\beta(\epsilon_k)}{2E_\mu^\beta(\epsilon_k)} \left(1 + \frac{S^2}{(E_\mu^\beta(\epsilon_k) - \tilde{\epsilon}_f)^2}\right) \tag{22}$$

where $2\tilde{\epsilon}_f$ marks the bottom of the upper band.

Upon evaluation, we get the following

$$I_4 = \frac{1}{2}\left[\ln\left|\frac{\Theta}{2\tilde{\epsilon}_f}\right| + \frac{S^2}{\tilde{\epsilon}_f^2}\left(\ln\left|\frac{\Theta}{2(\Theta - \tilde{\epsilon}_f)}\right| - \frac{\tilde{\epsilon}_f}{\Theta - \tilde{\epsilon}_f} + 1\right)\right]. \tag{23}$$

Combining these terms with those of equation (15), we obtain a new expression for $1/\rho_0 V_l$

$$\frac{1}{V_l} = \rho_0\left(\ln\left|\frac{\beta_c}{2}\Lambda'\right| + 1\right) + \rho_*\left(\ln\left|\frac{\beta_c}{2}\Omega'\right| + C'\right) \tag{24}$$

where C' is given by

$$C' = \frac{1}{2}\left(\frac{(N-1)}{n_f} + 1 - \frac{2\tilde{\epsilon}_f^2}{\Theta^2 - \tilde{\epsilon}_f^2}\right). \tag{25}$$

Here Λ' is $\Theta\delta'$ where δ' is simply equal to $\delta/\sqrt{2}$ and $\Omega'$is given by

$$\Omega' = \Theta\tilde{\epsilon}_f\sqrt{\frac{\delta_l(N-1)}{2n_f(\Theta^2 - \tilde{\epsilon}_f^2)}} \tag{26}$$

in the limit $\tilde{\epsilon}_f/\Theta \ll 1$.

As before, we consider the two expressions for the superconducting transition temperature that now arise. Thus, after expanding, we get

$$T_c = \left(\frac{\exp(1)}{2}\right)\Lambda' \exp\left(-\frac{1}{\rho_0 V_l}\right) \tag{27}$$

$$T_c = b'\Omega' \exp(-1/\rho_* V_l) \tag{28}$$

where b' is $\exp(C')$.

We see that the resulting expressions are again consistent with the numerical analysis. The inclusion of the upper-band term has very little effect upon the final expressions. The second enhanced form will dominate for HF systems as before.

7. Discussion

We have derived an expression for the superconducting transition temperature of an HF system using a simple weak coupling approach. This is found to be independent of the cut-off energy, Θ, provided it is large compared with the Kondo energy. Instead, we find that the Kondo energy takes on the role of cut-off and naturally arises in equations (22) and (29), which define the transition temperature with and without the inclusion of an upper-band term. In this way we see that the fundamental structure of the heavy-quasiparticle bands, which characterizes HF systems, manifests itself quite clearly within this basic theory of superconductivity.

The inclusion of the upper-band term has in fact little effect upon the outcome. It shows that incorporating the heavy-quasiparticle band structure in such a way so as to include the scattering of a pair of quasiparticles from the lower to upper band does not change the basic results of this theory.

The size of the pairing interaction between quasiparticles required to yield a transition temperature of the observed value for UPt_3 is two orders of magnitude smaller than that expected for a conventional superconductor. This presumably derives from the fact that it is the enhanced density of states that enters into the exponent in expressions (22) and (29), and so, the interaction may be smaller than usual.

There are, of course, many questions that still remain, but perhaps the three most urgent ones to be dealt with for this particular model are:

(1) What effect on this type of theory will the inclusion of quasiparticle interactions have?

(2) How would the proper treatment of the underlying antiferromagnetism affect these results?

(3) Will the introduction of a microscopically derived structure for the interactions, V_l, reveal a preferred pairing symmetry within this model?

We shall address these issues in a later publication.

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